

## BCA/D-19

## MATHEMATICAL FOUNDATION-I

## Paper-BCA-113

Time allowed : 3 hours]

[Maximum marks : 80

Note : Attempt five questions in all, selecting one question from each section. Question No. 1 is compulsory.

1. (a) If  $A = \{1, 2, 3, 4\}$   
 $B = \{2, 4, 6\}$   
 $C = \{1, 2, 5\}$   
 $U = \{x : x \text{ is a positive integer } < 9\}$   
 then find  
 $(A' - B') \cap (B' - C')$  4
- (b) If  $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$ ; find x 3
- (c) Find the derivative of  $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$  w.r.t. x 3
- (d) Prove that  $y = A \cos x + B \sin x$  is a solution of the differential equation  $\frac{d^2y}{dx^2} + y = 0$  3
- (e) Solve the differential equation  $2\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 7\frac{dy}{dx} - 2y = 0$  3

## Section-I

2. (a) In a class of 60 boys, there are 45 boys who play cards and 30 boys who play carrom. Also each boy like to play atleast one game. Use set operations to find:
- (i) How many boys play both the games ?
- (ii) How many play cards only ?
- (iii) How many play carrom only ? 8
- (b) If  $A = \{1, 2, 3, 4\}$ ,  $B = \{3, 4, 5, 6\}$ ,  
 $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$   
 Verify that : (i)  $A - B = A \cap B' = B' - A'$   
 (ii)  $(A \cup B)' = A' \cap B'$  8
- (a) Let  $A = \{1, 2, 3, \dots, 45\}$  and R is a relation "is square of" in A. Write R as a subset of  $A \times A$  and find domain and Range of R. 8
- (b) How many arrangements can be made of the letter of the word "ARRANGEMENT". In how many of these the vowels are together ?

## Section-II

- (a) Using  $\epsilon - \delta$  definition, prove that

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$$

8

(b) If  $y = \cos^{-1} \left[ \frac{2 \cos x + 3 \sin x}{\sqrt{13}} \right]$ ; prove that

$$\frac{dy}{dx} = -1 \quad 8$$

5. (a) If  $e^x + e^y = e^{x+y}$ ; prove that  $\frac{dy}{dx} = -e^{y-x}$  8

(b) Differentiate  $x^{\sin^{-1} x}$  w.r.t.  $\sin^{-1} x$  8

### Section-III

6. (a) Find the differential equation of all the circles of radius  $a$ . 8

(b) Solve the differential equation  
 $(1 + y^2) dx = (\tan^{-1} y - x) dy$  8

7. (a) Solve the differential equation  
 $(1 + e^{xy}) dx + e^{xy} \left( 1 - \frac{x}{y} \right) dy = 0$  8

(b) Solve the following differential equation.  
 $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$  8

### Section-IV

8. (a) Solve the differential equation  
 $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$  8

(4)

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + 4y = x \sin x \quad 8$$

9. (a) Solve the differential equation

$$x \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = \frac{1}{x} \quad 8$$

(b) Solve the differential equation

$$(x+1)^2 \frac{d^2y}{dx^2} + (x+1) \frac{dy}{dx} = (2x+3)(2x+4) \quad 8$$

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